CALIFORNIA INSTITUTE OF TECHNOLOGY

EARTHQUAKE ENGINEERING RESEARCH LABORATORY

ON THE STATISTICS AND POSSIBLE TRIGGERING MECHANISM OF EARTHQUAKES IN SOUTHERN CALIFORNIA

BY

MIHAILO D. TRIFUNAC

EERL 70-03

A REPORT ON RESEARCH CONDUCTED UNDER A GRANT FROM THE NATIONAL SCIENCE FOUNDATION

PASADENA, CALIFORNIA

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ERRATA

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24	5 from bottom	Equation should be $F_D^k(t) = f(t)/a_k$
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ABSTRACT

A statistical analysis is presented for the Southern California region, for 33 years beginning on January 1, 1934, and for all earthquakes with Richter magnitude ≥ 3.0.

A relatively general model for triggering shocks is described which admits any perturbation function which is stationary in time and has a power spectral density. The tides are considered as possible admissible perturbations, and it is shown that on the basis of the earthquake data and the model considered, they may be accepted as statistically possible triggering mechanisms for earthquakes in Southern California.

INTRODUCTION

The statistics of the occurrence as well as the generating mechanisms of the earthquakes, as events in time, has attracted the attention of many investigators. The first objective of this study is to bring up to date some aspects of the statistical analysis of earthquake occurrence for the Southern California region. This area is covered by one of the best and oldest station networks in the world.

The second objective is to investigate the possibility of explaining the observed statistical behavior of earthquake occurrences in time as a consequence of a simple perturbation model. This approach has already been considered in many previous investigations, but the models that have been used or the methods of the analysis have been different from the present approach.

HISTORICAL REVIEW

The idea of small perturbations superimposed on the main driving forces which eventually cause earthquakes is not new. Early attempts to study the periodicities in earthquake occurrence were stimulated directly or indirectly by this idea.

One of the first detailed treatments of the subject was by Davison (1938). He found periods of 42 minutes, 24 hours, 14.8 days, 29.6 days, 6 months, 1 year, 11 years, and 19 years in the earthquake data available at this time. All these periods with the exception of the shortest one of 42 minutes were considered to be the consequence of some astronomical phenomenon.

The majority of past investigations can be grouped into two classes. The authors from the first group treated the earthquake data from the point of view of periodogram analysis, trying to obtain hidden periodicities which were then correlated with known astronomical or geophysical phenomena. Early examples of these are Knott (1886), Fuchs (1886), Montesus de Ballore (1889), Davison (1938), Cancani (1901), Omori (1902), Allen (1936), Jeffreys (1938), Perine (1949) and others.

The investigators of the second group attempted to explain earthquake events in time or space as the consequence of a mechanism which was either simple enough for mathematical formulation or could be directly correlated with the existing earthquake data. Some recent studies belonging to this second group follow.

Stetson (1935) correlated the times of the occurrence of deep focus earthquakes with the lunar hour and declination and stated that these were in good agreement.

Variations in the sea level producing periodic loading of the continental shelves was discussed by Leypoldt (1941) as a possible triggering mechanism. In addition to tidal variations, Leypoldt also attempted correlations with seasonal thermal expansion of the oceans and to the release of snow, ice and ground water.

McMurry (1941) analyzed some 320 deep-focus earthquakes, testing for periods of 6, 12 and 14 months and for correlation with lunar and solar hour angles. He found that "none of these results could be interpreted as other than accidental". He then proceeded to analyze the effects of the earth tides and oceanic tidal loads, and concluded that the "only stress changes that appear to warrant study with reference to their effect on earthquakes are those due to earth tides."

Periodicities in earthquake occurrence as a consequence of the wandering of the earth's axis of rotation was proposed by Landsberg (1940). He suggested that motion of air masses due to climatological and astronomical changes influences the mass-distribution on the Earth which then in turn "results in a change of position of the Earth's poles of rotation". He concludes that "the rather high correlation coefficient found between earthquake-numbers and hemispherical pressure changes places the latter ones in the class of a very important trigger forces, at least for the decade 1921-1930".

Van den Dungen et. al, (1952) considered the correlation between irregularities in the speed of earth rotation and seismicity. They used the mean annual variations in the speed of the rotation of the earth calculated by Stoyko (1950). These variations represent a correction to be superimposed on the mean speed of rotation calculated for an interval of five years. Equivalently, this correction can be thought of as a variation in the length of a day (l.o.d.) with respect to the mean length of the day during five years. Van den Dungen and coworkers compared this variation of l.o.d. with the annual frequency of the earthquakes in the British Islands and the annual frequency of earthquakes registered at Uccle (Belgium) between 1910 and 1945 and they conclude: —S'il est permis d'interpreter ces donnés peu étendues et peu fournis, mais cependant assez homogènes, il apparait que les tremblements de terre seraient le plus frequentes lors de "changements du signe de la marché de l'horloge terre". —

Cox, Van den Dungen and Van Mieghem (1951) have also considered correlation of the annual variation in the length of the day to the annual frequency of the earthquakes registered during 35 year period at Uccle, with apparently an excellent agreement.

Lamakin (1966) found that the Baikal earthquakes had strong periodicites of 18.6 years. All the earthquakes with the epicenters on the north-western side of and on the Baikal platform occurred during the high lunar culmination. On the other hand all shocks on the eastern side of the Baikal platform occurred during the low lunar culmination.

Difficulties in the Fourier analysis of the times of earthquake occurrence and the significance of the resulting amplitudes have been discussed, among others, by Jeffreys (1938) and Knopoff (1964). To avoid such difficulties, Knopoff correlated the earthquake origin times in real time to the relative tidal time calculated with respect to the maximum tide. The data that were used in this work were shocks recorded by the Pasadena network for period from 1934 to 1957 and magnitudes ≥ 2.0. If tidal forces were indeed the triggering forces, it was expected that this approach would indicate the tendencies of earthquakes to occur clustered close to some characteristic tidal times associated with the critical stress or strain in the particular fault region. However, from the results of these correlations, Knopoff concludes "that the largest possible triggering mechanism in the earth, namely that of oscillatory tidal strains has no detectable influence upon the times of occurrence of small earthquakes in Southern California".

The purpose of this brief historical review has been to point out some of the interesting and possibly relevant features of the prior work in this field as it may be related to this particular study. It has not been meant to be complete nor exhaustive. A detailed and up to date review of some other aspects of statistical analysis and prediction of earthquakes may be found in the recent paper by Lomnitz (1966).

FREQUENCY ANALYSIS OF SOUTHERN CALIFORNIA EARTHQUAKES

The data on the sequence of seismic events used in this study are based on the Quarterly Bulletin of Local Shocks of the Seismological Laboratory of the California Institute of Technology. The Bulletin, issued regularly since January 1, 1934, gives all pertinent information regarding shocks of Richter magnitude 3.0 and greater, for the area termed the "Southern California region" (Nordquist 1964) Fig. 1. Data from all earthquakes between January 1, 1934 and December 31, 1967 were stored on IBM cards and on magnetic tape.

In order to arrange the data in a simple form suitable for analysis, a sequence was formed such that each event was given a time coordinate t_i in units of days, relative to January 1, 1934. This was done in the following way:

$$t_i = (\text{year}_i - 1934)365.256 + (\text{month}_i - 1)30.438 + \text{day}_i$$
$$+ \text{hour}_i/24.00 + \text{min}_i/1440.00 \qquad (1)$$

It is seen that this simplifying scheme neglects the fact that the calendar year has 365.00 days and each fourth year has 366.00 days, and also that the number of days in the month is not a constant but may be 28, 29, 30, or 31 days. It was felt, however, that this re-arrangement of the data would be of negligible influence on the results. A one dimensional array of monotonically increasing time coordinates

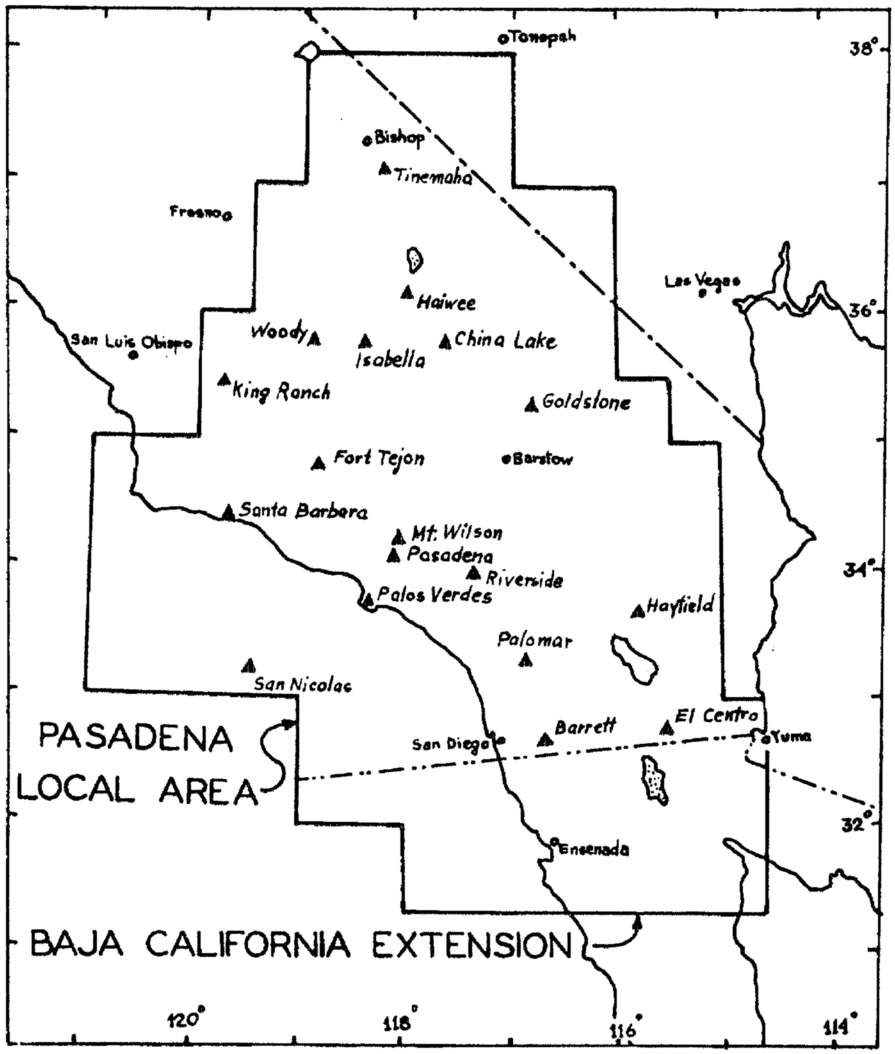

Figure 1. The map of the area covered by the Pasadena seismological network, showing locations of stations operating on January 1, 1963 (Allen et al. 1965).

t_i was obtained ranging from 0.000 to 12418.194 days, corresponding to the 33 years that were considered in this analysis, and consisting of 7459 events of Richter magnitude ≥3.0. This basic sequence of data was further subdivided into the following groups

M ≥3.5	with total	3736	shocks or	50.00%
M ≥4.0	"	1840	"	24.65%
M ≥4.5	"	747	"	10.01%
M ≥5.0	"	230	"	3.80%

All earthquakes of magnitude less than 3.0, reported in the Bulletin in earlier years, were disregarded in this analysis. Shocks that occur within the area shown in Fig. 1, but south of the international border in Mexico, are listed in the Bulletin only if they are greater than, or equal to, 4.5 on the Richter magnitude scale.

Knopoff (1964) analyzed seismic events between 1934 and 1957 for a somewhat smaller region of Southern California using the same original data. He divided the twenty-four year time interval into a sequence of decades (intervals of ten days each) and calculated the number of decades which had 0, 1, 2 . . . n events, in order to compare the histogram with the Poisson distribution.

It was felt that a similar analysis, but more detailed, would be useful for many applications in earthquake engineering, earthquake statistics and studies related to earthquake triggering mechanisms.

A procedure essentially the same as that of Knopoff (1964) was adapted and the 33 year period subdivided into equal ten day intervals. The number of earthquakes in each decade was tabulated. After the first such calculation was completed, the origin time of the first decade was shifted by one day and the analysis repeated in exactly the same way. This shifting of the origin time of the decade sequence was repeated ten times, each followed by the tabulation of a histogram similar to that shown in Fig. 9. The ten histograms were assumed to be equally probable and averaged to give the final histogram for all shocks greater than, or equal to, 3.0, 3.5, 4.0, 4.5 and 5.0 on the magnitude scale. Histograms associated with each of these groups are given in a tabulated form in Tables 1 to 5. As can be seen in those tables, coordinates of the histograms are decimal rather than integer numbers as a consequence of the averaging procedure. Foreshocks, aftershocks and earthquake swarms are present in the above sequences.

It would be difficult, if not impossible, to separate foreshocks, aftershocks, or swarms from the complete sequence of the events belonging to any of the magnitude groups. Any scheme of elimination would have to be based on some kind of model describing the effect of triggering interactions among closely spaced faults, and thus would not be generally acceptable. To avoid this difficulty in the physical formulation of the problem, a rather simple procedure was adopted, as proposed by Knopoff (1964). All decades in the original sequence ($M \geq 3.0$) that had more than 13 events were rejected. Furthermore, the decades preceding and following the one with 13 events, or more, were also rejected from the analysis.

Table 1. Earthquakes of Southern California for 1934 to 1967 Period

M = 3.0 and Greater ; 1 Decade = 10 Days

Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades
0	44.20	43	0.10	86	0.	129	0.
1	92.90	44	0.10	87	0.10	130	0.
2	144.40	45	0.	88	0.10	131	0.
3	166.40	46	0.30	89	0.	132	0.
4	179.60	47	0.20	90	0.	133	0.
5	146.30	48	0.20	91	0.	134	0.
6	109.60	49	0.20	92	0.	135	0.
7	86.20	50	0.20	93	0.10	136	0.
8	66.50	51	0.60	94	0.10	137	0.
9	44.80	52	0.20	95	0.	138	0.
10	30.30	53	0.30	96	0.	139	0.
11	24.80	54	0.50	97	0.10	140	0.10
12	13.90	55	0.50	98	0.	141	0.
13	12.00	56	0.30	99	0.	142	0.
14	10.50	57	0.50	100	0.10	143	0.
15	8.90	58	0.20	101	0.	144	0.
16	7.60	59	0.10	102	0.10	145	0.
17	5.50	60	0.10	103	0.	146	0.
18	5.80	61	0.	104	0.	147	0.
19	3.30	62	0.10	105	0.	148	0.10
20	4.50	63	0.	106	0.	149	0.
21	2.20	64	0.	107	0.	150	0.
22	3.00	65	0.10	108	0.	151	0.
23	2.30	66	0.	109	0.	152	0.
24	1.70	67	0.10	110	0.	153	0.
25	1.30	68	0.10	111	0.	154	0.
26	0.50	69	0.10	112	0.	155	0.
27	2.00	70	0.	113	0.	156	0.
28	1.20	71	0.10	114	0.	157	0.
29	1.00	72	0.20	115	0.10	158	0.10
30	0.70	73	0.	116	0.	159	0.
31	1.00	74	0.	117	0.	160	0.
32	0.50	75	0.40	118	0.10	161	0.
33	0.30	76	0.10	119	0.	162	0.10
34	0.80	77	0.10	120	0.	163	0.
35	0.90	78	0.30	121	0.	164	0.
36	0.80	79	0.10	122	0.10	165	0.
37	1.20	80	0.10	123	0.	>165	0.
38	0.30	81	0.10	124	0.		
39	0.20	82	0.40	125	0.		
40	0.40	83	0.20	126	0.		
41	0.	84	0.10	127	0.10		
42	0.60	85	0.20	128	0.		

Table 2. Earthquakes of Southern California for 1934 to 1967 Period

M = 3.5 and Greater ; 1 Decade = 10 Days

Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades
0	231.50	43	0.10	86	0.	129	0.
1	288.20	44	0.20	87	0.	130	0.
2	250.20	45	0.20	88	0.	131	0.
3	166.80	46	0.40	89	0.	132	0.10
4	105.90	47	0.10	90	0.	133	0.
5	60.20	48	0.30	91	0.	134	0.
6	34.10	49	0.20	92	0.	135	0.
7	27.60	50	0.10	93	0.	136	0.
8	15.90	51	0.60	94	0.10	137	0.
9	9.90	52	0.30	95	0.	138	0.
10	6.20	53	0.	96	0.	139	0.10
11	4.00	54	0.40	97	0.	140	0.
12	2.90	55	0.	98	0.10	141	0.
13	4.00	56	0.20	99	0.	142	0.
14	3.70	57	0.20	100	0.	143	0.
15	3.90	58	0.10	101	0.	144	0.
16	2.50	59	0.10	102	0.	145	0.
17	2.40	60	0.	103	0.	146	0.
18	3.00	61	0.	104	0.	147	0.
19	1.50	62	0.30	105	0.	148	0.
20	1.00	63	0.	106	0.	149	0.10
21	0.70	64	0.	107	0.	150	0.
22	1.30	65	0.	108	0.	151	0.
23	0.70	66	0.10	109	0.	152	0.
24	0.70	67	0.	110	0.10	153	0.10
25	0.80	68	0.	111	0.	154	0.
26	0.60	69	0.20	112	0.	155	0.
27	0.70	70	0.	113	0.10	>155	0.
28	0.50	71	0.10	114	0.10		
29	0.40	72	0.10	115	0.		
30	0.20	73	0.	116	0.		
31	0.40	74	0.	117	0.		
32	0.30	75	0.10	118	0.		
33	0.20	76	0.10	119	0.		
34	0.40	77	0.	120	0.		
35	0.30	78	0.	121	0.		
36	0.40	79	0.	122	0.10		
37	0.	80	0.	123	0.		
38	0.10	81	0.	124	0.		
39	0.10	82	0.	125	0.		
40	0.	83	0.	126	0.		
41	0.30	84	0.	127	0.		
42	0.10	85	0.	128	0.		

Table 3. Earthquakes of Southern California for 1934 to 1967 Period.

M = 4.0 and Greater ; 1 Decade = 10 Days

Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades
0	555.40	43	0.	86	0.	129	0.
1	342.60	44	0.	87	0.	130	0.10
2	172.80	45	0.10	88	0.	>130	0.
3	66.10	46	0.	89	0.10		
4	32.90	47	0.	90	0.		
5	19.90	48	0.	91	0.		
6	8.70	49	0.	92	0.		
7	6.60	50	0.	93	0.10		
8	7.00	51	0.40	94	0.		
9	4.50	52	0.	95	0.		
10	2.50	53	0.	96	0.		
11	2.40	54	0.10	97	0.		
12	1.50	55	0.	98	0.		
13	2.60	56	0.	99	0.		
14	2.80	57	0.10	100	0.		
15	2.30	58	0.	101	0.10		
16	0.90	59	0.	102	0.		
17	0.50	60	0.	103	0.		
18	0.50	61	0.	104	0.		
19	0.90	62	0.10	105	0.10		
20	1.20	63	0.	106	0.		
21	0.10	64	0.	107	0.		
22	0.10	65	0.	108	0.		
23	0.30	66	0.	109	0.		
24	0.10	67	0.	110	0.		
25	0.20	68	0.	111	0.		
26	0.10	69	0.10	112	0.		
27	0.60	70	0.	113	0.10		
28	0.	71	0.	114	0.		
29	0.10	72	0.10	115	0.		
30	0.40	73	0.	116	0.		
31	0.30	74	0.	117	0.		
32	0.	75	0.10	118	0.		
33	0.	76	0.10	119	0.		
34	0.40	77	0.10	120	0.10		
35	0.20	78	0.	121	0.		
36	0.10	79	0.	122	0.10		
37	0.	80	0.	123	0.		
38	0.10	81	0.	124	0.		
39	0.	82	0.	125	0.		
40	0.20	83	0.	126	0.10		
41	0.	84	0.	127	0.		
42	0.	85	0.	128	0.		

Table 4. Earthquakes of Southern California for 1934 to 1967 Period

M = 4.5 and Greater ; 1 Decade = 10 Days

Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades	Earth. per Decade	Number of Decades
0	889.20	43	0.				
1	234.00	44	0.10				
2	55.30	45	0.				
3	18.60	46	0.10				
4	7.10	47	0.10				
5	6.60	48	0.10				
6	2.60	49	0.				
7	4.30	50	0.30				
8	3.00	51	0.10				
9	0.70	52	0.20				
10	1.20	53	0.				
11	1.00	54	0.10				
12	0.50	55	0.10				
13	1.20	56	0.20				
14	0.30	57	0.				
15	0.20	58	0.				
16	0.	59	0.				
17	0.	60	0.				
18	0.	61	0.				
19	0.10	62	0.				
20	0.30	63	0.				
21	0.	64	0.				
22	0.20	65	0.				
23	0.40	66	0.				
24	0.20	67	0.				
25	0.20	68	0.10				
26	0.	69	0.				
27	0.	70	0.				
28	0.	71	0.10				
29	0.20	72	0.				
30	0.10	73	0.				
31	0.20	74	0.				
32	0.10	75	0.10				
33	0.10	76	0.10				
34	0.	77	0.				
35	0.10	78	0.				
36	0.	79	0.				
37	0.	80	0.				
38	0.	>80	0.				
39	0.						
40	0.						
41	0.						
42	0.						

Table 5. Earthquakes of Southern California for 1934 to 1967 Period

M = 5.0 and Greater ; 1 Decade = 10 Days

Earth. per Decade	Number of Decades
0	1093.20
1	106.10
2	14.90
3	5.20
4	3.40
5	1.20
6	0.50
7	2.30
8	0.90
9	0.
10	0.10
11	0.10
12	0.40
13	0.10
14	0.40
15	0.30
16	0.10
17	0.20
18	0.
19	0.
20	0.10
21	0.30
22	0.
23	0.
24	0.
25	0.
>25	0.

Table 6. Earthquakes of Southern California for 1934 to 1967 Period

M = 3.0 and Greater ; 1 Decade = 10 Days

Foreshocks, Aftershocks and Swarms Eliminated

Earth. per Decade	Number of Decades
0	42.09
1	85.32
2	133.40
3	153.39
4	161.31
5	134.39
6	100.01
7	78.81
8	58.40
9	37.40
10	23.61
11	19.89
12	9.90
13	8.90

The remaining sequence of decades was then analyzed in exactly the same way as the original complete sequence. The ten histograms were averaged as before, and the results are given in Table 6 and Fig. 9.

It should be pointed out here that the decision to reject all decades with more than 13 events is quite arbitrary and has no physical basis. It is believed, however, that on the average this procedure probably eliminates most of the aftershocks and swarms in the sequence of $M \geq 3.0$. This is based on the assumption that a large number of earthquakes are the consequence of some system of driving forces which depend, among other things, on space and time coordinates, but are not interrelated among themselves through stresses or strains resulting from events that have already occurred. It is quite possible to imagine another extreme in which all earthquakes would be thought of as foreshocks or aftershocks of all other events. Such a point of view would, of course, make a model of the present type meaningless.

A more detailed future study of the frequency of occurrence of shocks in Southern California Region might call for several changes in the present method of data compilation. First, the exact division into 10 day intervals could be used instead of the simplifying transformation (1). Second, the present scheme of eliminating the aftershocks by rejecting all decades with more than 13 events is certainly crude and does nothing in cases when one or two aftershocks occur in each decade. Finally, the area south of the

Mexican border could be accounted for, since for that region the Bulletin of Local Shocks lists only the earthquakes with $M \geq 4.5$. This area represents only about 15% of the total Southern California Region covered by the Pasadena Seismological network (Figure 1). The above mentioned possible future improvements, with the exception of the aftershock elimination scheme would however introduce only very minor changes in the histograms reported here and were not considered necessary for the present study.

PERTURBATION MODEL FOR TRIGGERING EARTHQUAKES

There may be several reasons for difficulty in the studies of earthquake triggering based on Fourier analysis or the correlation in actual time. It may be that the rocks do not fracture instantaneously when a critical stress level is attained for the short time corresponding to the rapid oscillatory perturbing forces. It is also possible that the fracture occurs in some cases after a short period of accelerated creep. If this period is on the average of the order of 12 hours, or greater, it may be sufficient to disturb any clustering that would otherwise be detectable. Finally, in many studies earthquake data are equally weighted in the spatial sense and no distinctions are made regarding the occurrence along specific faults or for the orientation of these faults. The effect of this is that corresponding times when critical stresses occur with respect to the perturbing maxima and minima of the perturbation forces, are randomly phase shifted according to the randomnes in fault orientation and this disturbs any possible correlation.

In an attempt to avoid the above mentioned difficulties a model is developed here that circumvents the necessity of the detailed consideration of fault orientation and geometry. Also it is believed that the characteristic time interval used (10 days), is sufficiently long to smooth out various effects caused by the creep and other short delaying mechanisms which are of the order of a few days.

In order to simplify the analysis only one "typical" k^{th} focus of possible earthquake energy release is being considered in the formulation of the problem. All definitions and rules postulated for that k^{th} focus will equally well apply to other similar foci distributed in the three-dimensional space of finite dimensions.

First, let $\underset{\sim}{F}^k(t)$ be the "resultant driver" which provides the main source of the energy which causes the earthquakes at the k^{th} location in the solid containing many such locations. It is being assumed here that this driver is a consequence of some, or of all, of the several possible mechanisms that may be considered as principal sources of energy.

Next, let it be assumed that it is possible to name all of the possible critical states at the k^{th} location in the solid that will be sufficient to cause a fracture which will then propagate over some bounded subregion of the material and thus cause an earthquake. Let this critical tensor at the k^{th} location be called $\underset{\sim}{F}_c^k$. Furthermore, let $F_D^k(t)$ be the difference of the norms of the critical tensor $\underset{\sim}{F}_c^k$ and driver tensor $\underset{\sim}{F}^k(t)$

$$F_D^k(t) \equiv \| \underset{\sim}{F}_c^k \|_{c_k} - \| \underset{\sim}{F}^k(t) \|_{b_k}$$

A unique real number, called the norm and designated by $\| \circ \|$ has been taken here to represent a generalized notion of distance between a vector and a null vector. (Isaacson and Keller, 1966).

Subscripts on the norms (c_k and b_k) indicate that the norms associated with the critical tensor and the driver tensor need not be the same and in general may change, for example, from one fault system to another depending, among other things, on the fault orientation and dimensions. Particular definitions of these norms have not been specified here and are left open. The reason for this is that there exist several possible criteria for the fracture of the material. For example, the criterion of the maximum shearing stress could be mentioned, among others, as a possible basis for the definition of norms. It may be seen that this form of definition remains general enough so that many of the existing criteria may be used at will.

It follows that when $F_D^k(t)$ becomes zero the critical state is realized and the earthquake occurs. It is supposed that this occurs without significant time lag. From the following analysis, it may be concluded that the random shifts in time, caused by different fault orientations, or the "relatively short" period of accelerated creep before the earthquake, observed in the field (e.g. Smith, and Wyss, 1968), would not significantly change essential features of this model. This is because the chosen unit of the time represented by one decade tends to smooth out all delaying mechanisms shorter than 10 days.

After the earthquake had occurred, the situation at the k^{th} location is suddenly drastically changed and $F_D^k(t)$ experiences a jump. Furthermore, some other locations, (say the l^{th} location) possibly close in space to the k^{th} location, where the earthquake has just occurred, are disturbed and may experience jumps in $F_D^l(t)$

which in some cases will lead to an earthquake at that 1^{th} location (Fig. 2) Whether the earthquake at location 1, and possibly some other ones, will occur because of the earthquake at the k^{th} location will clearly depend on the relative magnitudes of the jumps and on the amplitudes of $F_D^1(t)$ and the other functions immediately before the instant when jumps have occurred.

Using this representation, the time behavior of a class of tentative earthquake foci in a certain seismic area may be represented on the common diagram of Fig. 2. It is assumed that the "flux" of the $F_D^k(t)$ functions for this area is on the average fairly uniform. If no jumps were to occur, zero intersections of the $F_D^k(t)$ functions would happen independently. Under these conditions, the number of earthquakes (or with the assumptions made here, the number of the zero intersections) per given time interval would be Poisson distributed. This hypothesis tested by Knopoff (1964) was rejected on the basis of the data for 24 years of Southern California earthquakes.

For the proposed model for triggering earthquakes, it is assumed that some perturbation mechanism exists. At this point no specific interpretation of this mechanism will be given. It is only assumed that these perturbations may be represented by some continuous stationary random function. Let $\underset{\sim}{F}_p^k(t)$ be the tensor representation of these perturbations at the k^{th} location in the solid. Then one can define

$$^{p}F_D^k(t) \equiv \|\underset{\sim}{F}_c^k\|_{c_k} - \|\underset{\sim}{F}^k(t) + \underset{\sim}{F}_p^k(t)\|_{b_k}$$

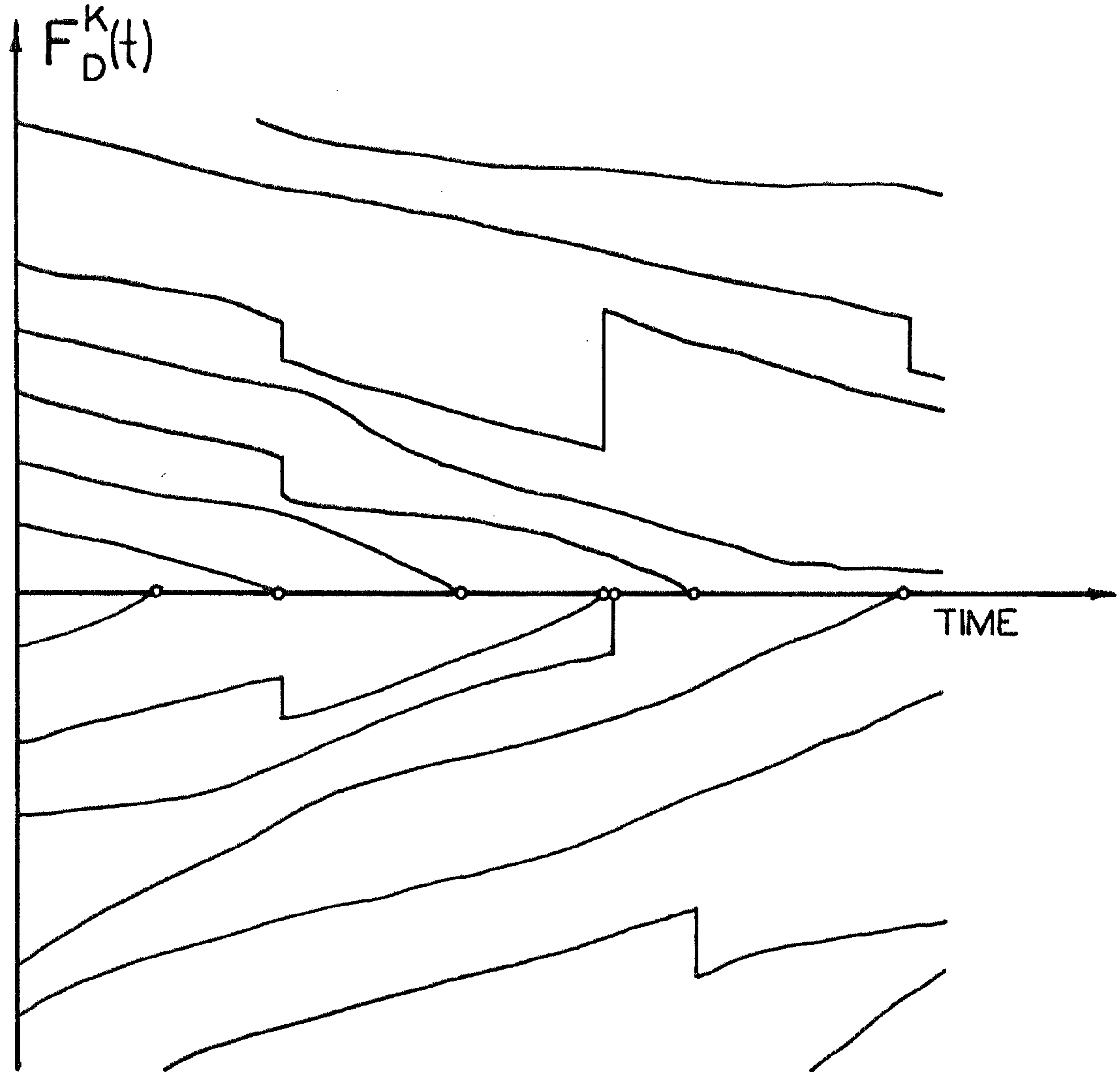

Figure 2. Functions $F_D^k(t)$ representing some of the faults in the seismic area considered for the model without the perturbation forces. Points on the zero intersections indicate times at which the earthquakes occur.

Further defining

$$f^k(t) \equiv \|\underset{\sim}{F}^k(t) + \underset{\sim}{F}_p^k(t)\|_{b_k} - \|\underset{\sim}{F}^k(t)\|_{b_k}$$

permits one to write

$$^pF_D^k(t) = \|\underset{\sim}{F}_c^k\|_{c_k} - \|\underset{\sim}{F}^k(t)\|_{b_k} - f^k(t)$$

Since

$$F_D^k(t) = \|\underset{\sim}{F}_c^k\|_{c_k} - \|\underset{\sim}{F}^k(t)\|_{b_k}$$

then

$$^pF_D^k(t) = F_D^k(t) - f^k(t)$$

This equation holds for the class of locations (say N locations) in the seismic region that is considered. Since the equation still holds if both left and right-hand side are normalized by some non-zero constant say a_k, we choose these constants such that

$$^pF_D^k(t) = F_D^k(t) - f(t)/a_k$$

Here it is assumed only that $a_k f^k(t) = a_m f^m(t) = f(t)$ independent of k and m.

Clearly, in the same way as for the model without the perturbations, the earthquake will occur when $^pF_D^k(t)$ becomes zero, or when

$$F_D^k a_k = f(t)/a_k$$

A graphical representation of this case is given in Fig. 3. It will be seen that most of the observations made about the model without the perturbation forces included will hold. However, the time at which the earthquake occurs is determined by the intersection of the

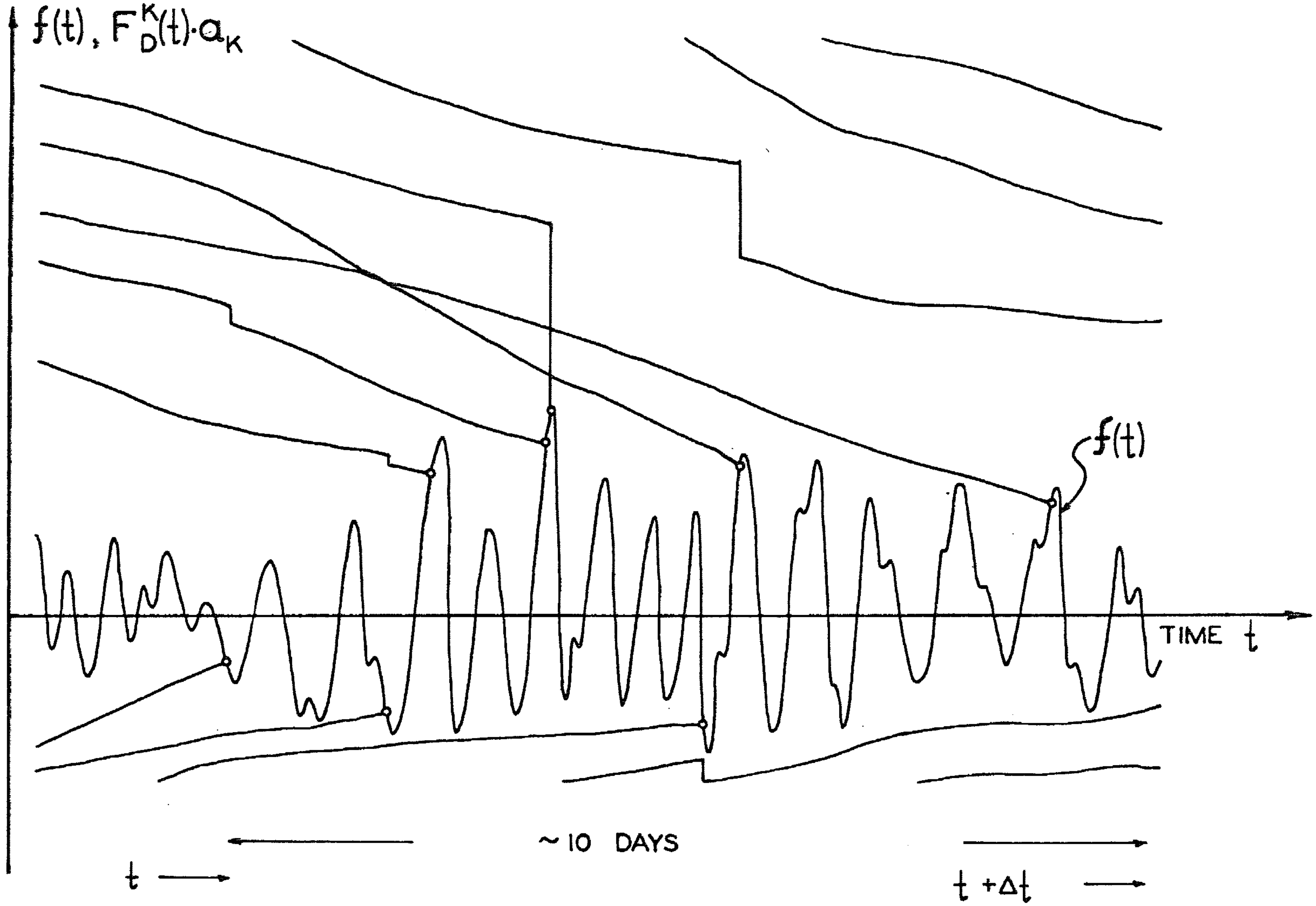

Figure 3. Functions $F_D^k(t) \cdot a_k$ and $f(t)$ representing some of the faults in the seismic area considered for the model with the perturbation forces. Points on the intersections of $F_D^m(t) \cdot a_m$ and $f(t)$ are indicating times at which the earthquakes occur.

$F_D^k(t)a_k$ and f(t), rather than with zero as before. Therefore, the statistical distribution of the earthquakes in time, even if the jump phenomena were eliminated, would not be Poisson distributed but would be determined by the properties of the f(t) function.

If it is assumed that $F_D^k a_k$ are on the average uniformly distributed in the t, $F_D^m(t)a_m$ plane in Fig. 3, then, disregarding all the earthquakes that are the consequence of the jump phenomena (aftershocks, foreshocks and swarms), it will be seen that the probability of an intersection occurring in the interval $(t, t+\Delta t)$ will be on the average the same as the probability of the maximum of f(t) lying above some critical level in the same interval of time. Statistical analysis of the earthquake occurrence, based on this hypothetical relationship, is attempted in the next section.

STATISTICAL DISTRIBUTION OF THE MAXIMA OF THE PERTURBATION FUNCTION AND THE NUMBER OF THE EARTHQUAKES IN A GIVEN INTERVAL OF TIME

As pointed out in the preceding section, under fairly general conditions it may be assumed that the probability density function of y earthquakes occurring in the fixed interval of time $(t, t+\Delta t)$ will be the same as the probability density function of the maxima of the function f(t). Knowledge of the functional relationship between the amplitude of f(t) and y the number of earthquakes is essentially a part of this assumption. The simplest possible relationship between y and the amplitude of f(t) is of course linear.

In order to derive the probability density distribution of y earthquakes in the fixed interval of time, some results on the distribution of the maxima of a random function are briefly outlined following Cartwright and Longuet-Higgins (1956).

Let f(t) be a continuous random function of one variable t. Furthermore, suppose that it may be represented by the series

$$f(t) = \sum_{n=0}^{\infty} c_n \cos(\sigma_n t + \epsilon_n)$$

where ϵ_n represent randomly and uniformly distributed phases in the interval between 0 and 2π. Amplitudes c_n are associated with the frequencies σ_n which are densely distributed in $(0, \infty)$ such that

$$\sum_{\sigma_n=\sigma}^{\sigma+d\sigma} \tfrac{1}{2} c_n^2 = E(\sigma)d\sigma$$

where $E(\sigma)$ is a given function called the energy spectrum or the power spectrum of f(t).

As usual, define the n^{th} moment of $E(\sigma)$ about the origin as

$$m_n = \int_0^\infty E(\sigma)\, \sigma^n d\sigma$$

and in particular

$$m_0 = \int_0^\infty E(\sigma)d\sigma$$

Next, the height ξ of a local extremum (crest) with respect to the mean height of the function f(t) is considered. It may be shown (Cartwright and Longuet-Higgins, 1956; Appendix I) that the distribution of the maxima of the ξ depends only on two parameters, $m_0^{1/2}$ and ϵ which is given by

$$\epsilon = \left(\frac{m_0 m_4 - m_2^2}{m_0 m_4}\right)^{1/2}$$

Let

$$\eta \equiv \frac{\xi}{m_0^{1/2}}$$

then the probability distribution of η becomes $m_0^{1/2}$ times the distribution of f(t)

$$p(\eta) = m_0^{1/2} p(\xi)$$

and (Cartwright and Longuet-Higgins, 1956; Appendix I):

$$p(\eta) = \frac{1}{(2\pi)^{1/2}}\left[\epsilon e^{-\frac{1}{2}\frac{\eta^2}{\epsilon^2}} + (1-\epsilon^2)^{1/2}\,\eta e^{-\frac{1}{2}\eta^2}\int_{-\infty}^{\eta\frac{(1-\epsilon^2)^{1/2}}{\epsilon}} e^{-\frac{1}{2}x^2}dx\right]$$

Since f(t) is statistically symmetrical about the mean level, it follows that the statistical distribution of the maxima is the reflection of the distribution of the minima in the mean level $\eta = 0$.

Graphs of the family of the distributions for the different values of ϵ are given in Fig. 4. When $\epsilon \to 0$ it is seen that the $p(\eta)$ distribution tends to the Rayleigh distribution, while when $\epsilon \to 1$ the distribution becomes Gaussian.

Let the cumulative probability $q(\alpha)$ be the probability of η exceeding the given value say α. Then

$$q(\alpha) = \int_{\alpha}^{\infty} p(\eta)d\eta .$$

Using the expression for $p(\eta)$ this can be written as

$$q(\alpha) = \frac{1}{(2\pi)^{1/2}}\left[\int_{\frac{\alpha}{\epsilon}}^{\infty} e^{-\frac{1}{2}x^2}dx + (1-\epsilon^2)^{1/2}\, e^{-\frac{1}{2}\alpha^2}\int_{-\infty}^{\alpha\frac{(1-\epsilon^2)^{1/2}}{\epsilon}} e^{-\frac{1}{2}x^2}dx\right]$$

Having outlined the results obtained by Cartwright and Longuet-Higgins, we now consider the application to our present problem.

Suppose that only a subclass of the whole population of the maxima which lie above a fixed level α is to be considered. We want

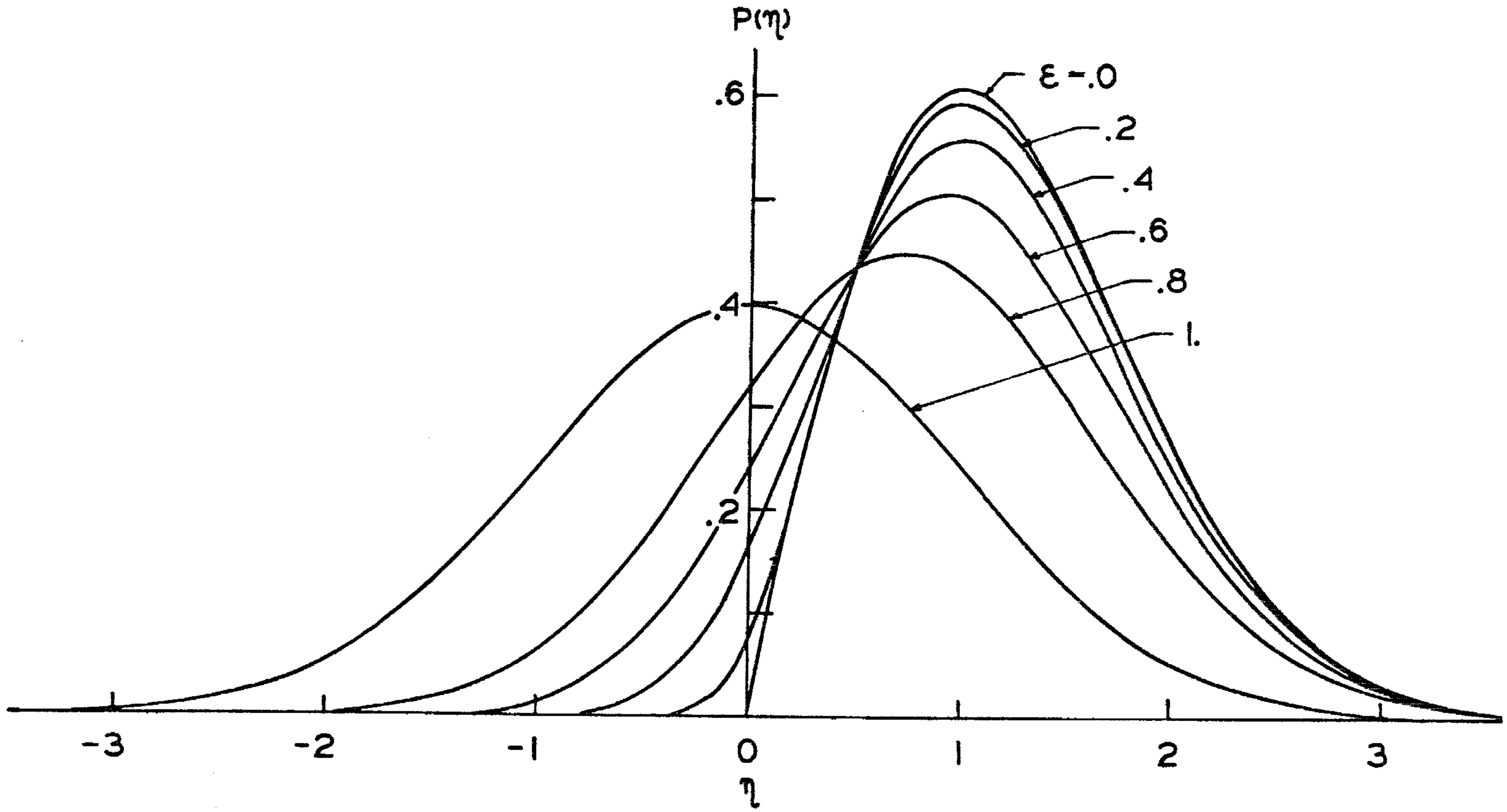

Figure 4. Graphs of $p(\eta)$ showing the probability distribution of the heights of maxima ($\eta = \xi/m_0^{1/2}$) for different values of the width ϵ of the energy spectrum (Cartwright and Longuet-Higgins, 1956).

to disregard the maxima that lie below. In order to arrive at the distribution of the maxima in that subclass, it suffices to observe that the probability density function will be proportional to the $p(\eta)$ distribution density function. The proportionality constant is determined so that the area under the probability density function is one. Then it follows that

$$p_\alpha(\eta) = p(\eta)/q(\alpha)$$

where $p_\alpha(\eta)$ is the probability density function of the maxima lying above level α. A family of distributions $p_\alpha(\eta)$ for the different values of ε and $\alpha = 0$ are shown in Fig. 5. It is seen that for $\varepsilon = 0$, the distribution is identical with $p(\eta)$ because for that case there are no negative maxima. For $\varepsilon = 1$, $p(\eta)$ (Fig. 4) is symmetrical about the mean level $\eta = 0$ showing that when the maxima are Gaussian distributed there are as many positive maxima as there are negative maxima. By inspection it can be seen that in this case $q(0) = 0.50$ and consequently $p_\alpha(\eta)$ is twice as big as $p(\eta)$ for the same value of ε. The distribution $p_\alpha(\eta)$ for the other values of ε lie in between those two extreme cases. For other values of α the behavior of $p_\alpha(\eta)$ is similar.

Let α be some yet unknown, fraction of $m_0^{1/2}$. Observing Fig. 3, it may be seen that the time gradient of $F_D^k(t) \cdot a_k$ will determine the possible value of α. For, if this gradient was close to zero, or very small, most of $F_D^k(t) \cdot a_k$ would tend to zero very slowly and only the highest maxima or minima of f(t) will have some chance of intersecting the curves $F_D^k(t) \cdot a_k$. This would lead to an α value relatively large and positive. On the other hand if $F_D^k(t) \cdot a_k$ functions were

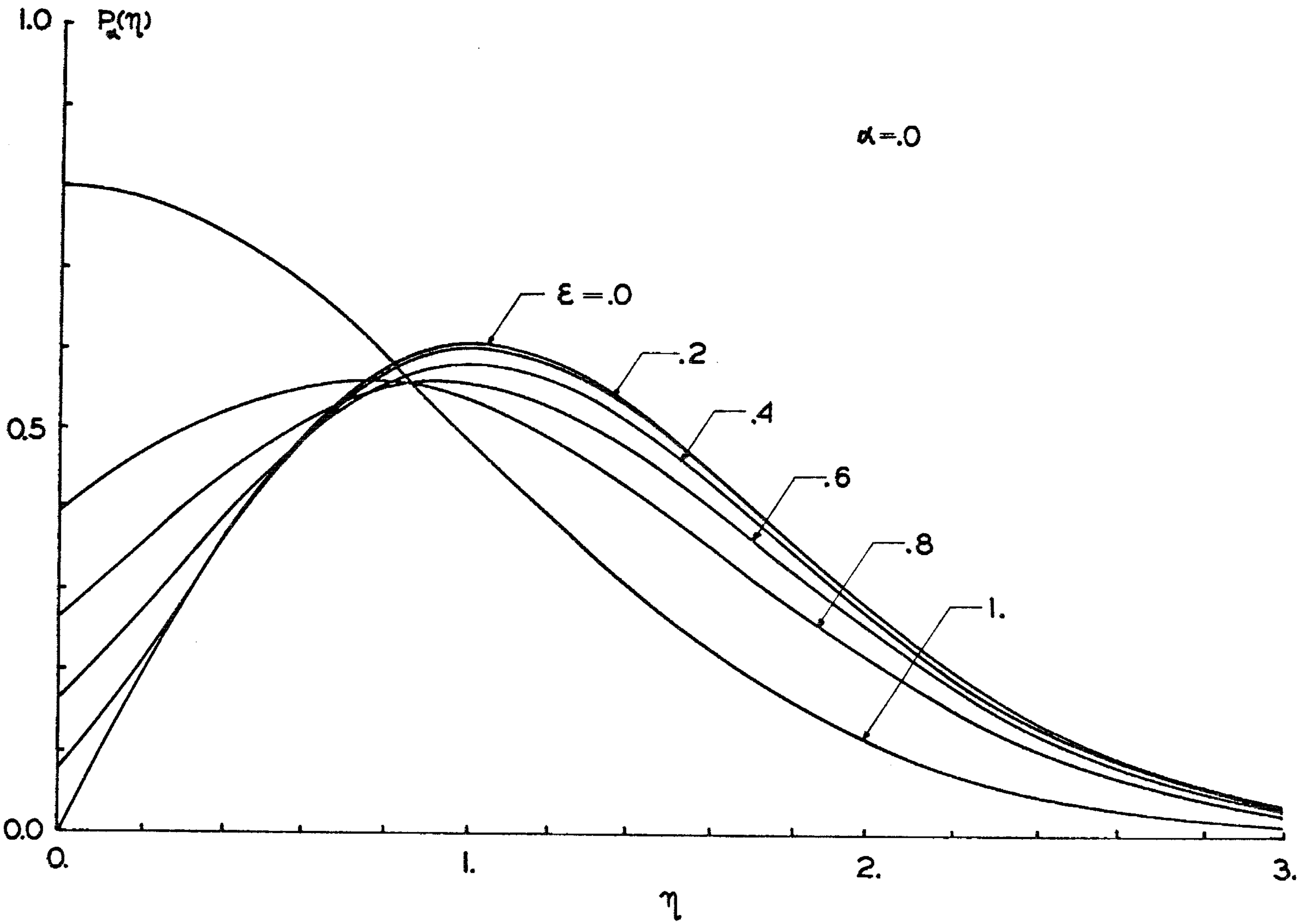

Figure 5. Graphs of $p_\alpha(\eta)$ showing the probability distribution of the heights of maxima ($\eta = \xi/m_0^{1/2}$) lying above the level α=0 for different values of ϵ.

steep and changing very rapidly, it might be expected that on the relative scale α would become smaller. From these imprecise but reasonable considerations, we assume here that

$$\eta = \alpha + \beta y$$

where β is the normalization constant. It may be recalled that $\eta = \xi / m_0^{1/2}$, so that η is a normalized height of the maxima of the function f(t). Then $1/\beta$ is a normalizing constant for y as $m_0^{1/2}$ is for ξ. Then one can write

$$p(y) = \beta p_\alpha(\eta)$$

Since

$$m_0^{1/2} p_\alpha(\xi) = \frac{1}{\beta} p(y)$$

$$p_\alpha(\eta) = m_0^{1/2} p_\alpha(\xi)$$

To test the hypothesis H_0, that the population density distribution function is p(y) against the alternative H_1 hypothesis, that the density distribution function of the population is not p(y), the chi-square test of goodness of fit is applied. It should be pointed out that the H_0 hypothesis is a simple alternative while H_1 is quite composite, because there is in general an infinite number of density distribution functions which are not p(y). Chi-square is given by

$$\chi^2 = \sum_{i=i}^{k} \frac{(f_i - np_{0_i})^2}{np_{0_i}}$$

where f_i is the frequency of the occurrence of E_i outcomes resulting from n independent trials and with total k possible different outcomes

E_1, E_2, ... so that $f_1+f_2+...+f_k=n$. If each outcome occurs with the probability p_i then χ^2 tests the hypothetical p_{0_i} probabilities which are the result of the H_0 hypothesis. If χ^2 is too large, i.e., $\chi^2>M$, the above test calls for the rejection of the hypothesis. If the probability of rejecting H_0 hypothesis, although it is a true one, is specified, M can be found from the χ^2 distribution tables with k-1 degrees of freedom. The family of the density distribution functions $p_\alpha(\eta)$ for all values of ϵ yielding p_{0_i} was used to calculate χ^2 for each set of given α,β and ϵ. The frequency occurrences f_i were taken from the histogram calculated (as described before) in such a way that every decade having more than 13 earthquakes in it was deleted, together with the immediately preceeding and following ones. The resulting histogram Fig. 9 was then modified so that the χ^2 test may be applied. Small amplitudes f_i that normally occur at the end of the histogram may make the χ^2 test unsatisfactory unless the class of data in that region are lumped together in some way. For this reason the decades having 10, 11, 12 and 13 events were grouped together into one class (Cartwright and Longuet-Higgins 1956, p. 227). This total of eleven amplitudes f_i were used to calculate χ^2 for each discrete set of parameters lying in the intervals $\alpha\in[-.50, .30]$, $\beta\in[.20, .30]$ and $\epsilon\in[.00, .80]$ with $k-1=10$ degrees of freedom and assuming that none of the parameters of the density distribution function $p_\alpha(\eta)$, i.e., α,β and ϵ, are inferred from the Table 6 or the histogram in Fig. 9. Statistical tables for the χ^2 distribution give (for 95% confidence level) ${}_{95}\chi^2_{10}=18.3$ which then is taken as the value of the constant M.

Results of these calculations may be seen in Fig. 6. Permissible values of α and β for which $\chi^2 \leq 18.3$ and for the given value of ϵ are enclosed by a series of ovals. If α or β, or both, were determined in some way from the original data, the number of degrees of freedom would be reduced accordingly, by one or two. In that case M=18.3 which yields contours in Fig. 6 would correspond to 96.8% and 97.9% confidence levels, respectively.

One would expect that the analysis of the intervals for the best choice of α, β and ϵ would suggest more detailed physical properties of the tentative candidates for the general perturbation function. If an assumption is made that the above boundaries are representative of the data analyzed, the question arises as to which values of α, β and ϵ are the best parameters for describing the original population. Unfortunately, the answer to this question can not be given uniquely. It depends on the criterion to be used in comparing several possible best choices of α, β and ϵ. If this criterion is chosen to be the minimum value of χ^2, one obtains $\alpha = 0.17$, $\beta = 0.24$, $\epsilon = 0$ and $\chi^2 = 3.875$. Another possible criterion may be the relative normalized width of the permissible values of α associated with the given value of ϵ and β, which can be calculated directly from Fig. 6. One would suppose that close to the best values of the parameters, permissible variations of any one of them alone would be the greatest. If all α widths are normalized w.r.t. the biggest one and plotted versus β for a set of ϵ values, the result can be seen in Fig. 7. This result suggests that possible values for ϵ might lie between .30 and .40 for β close to .25. Finally,

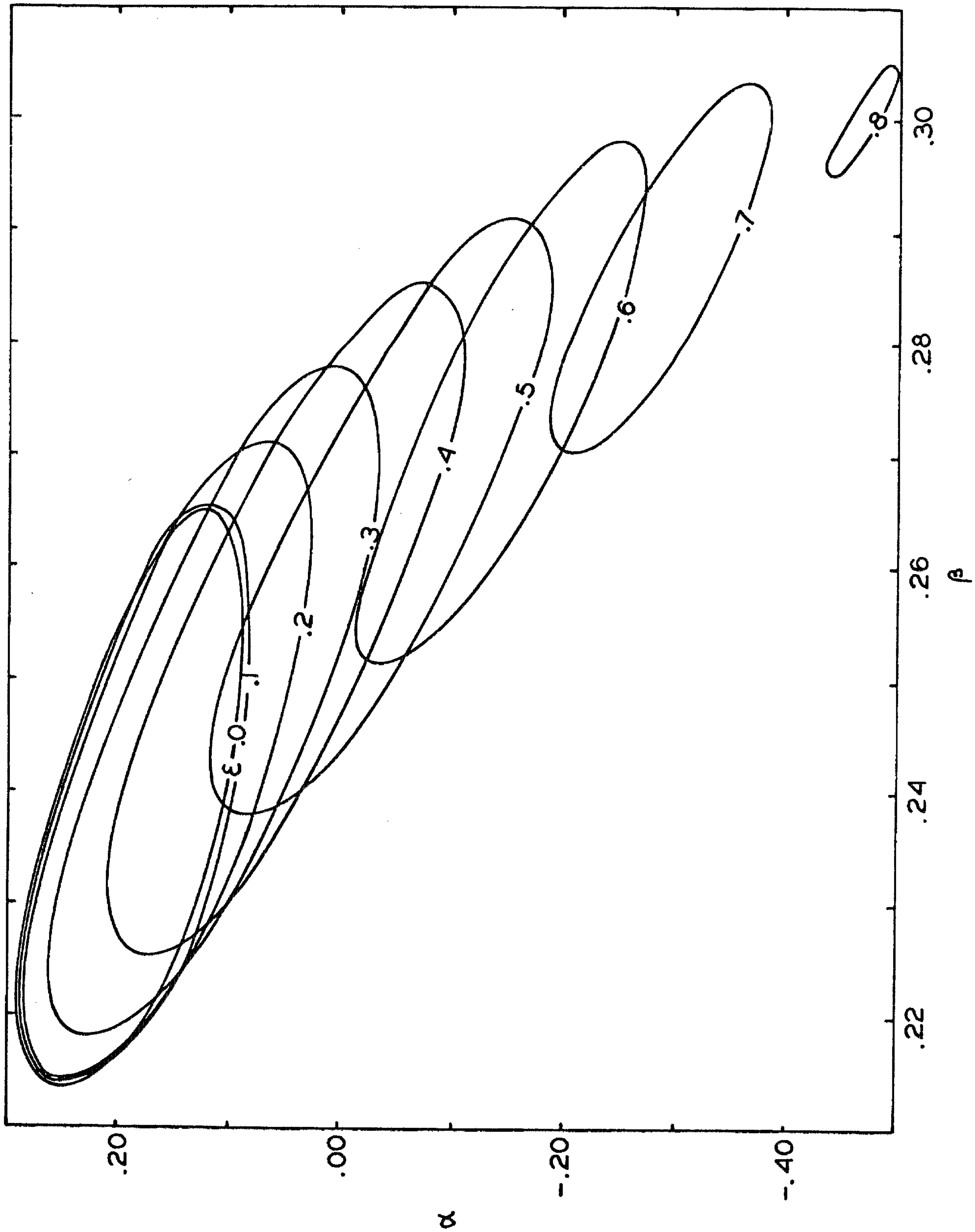

Figure 6. Graph showing the oval regions inside which for given values α, β and ε chi-square test permits the acceptance of H_o hypotheis ($\chi^2_{.95}$= 18.30).

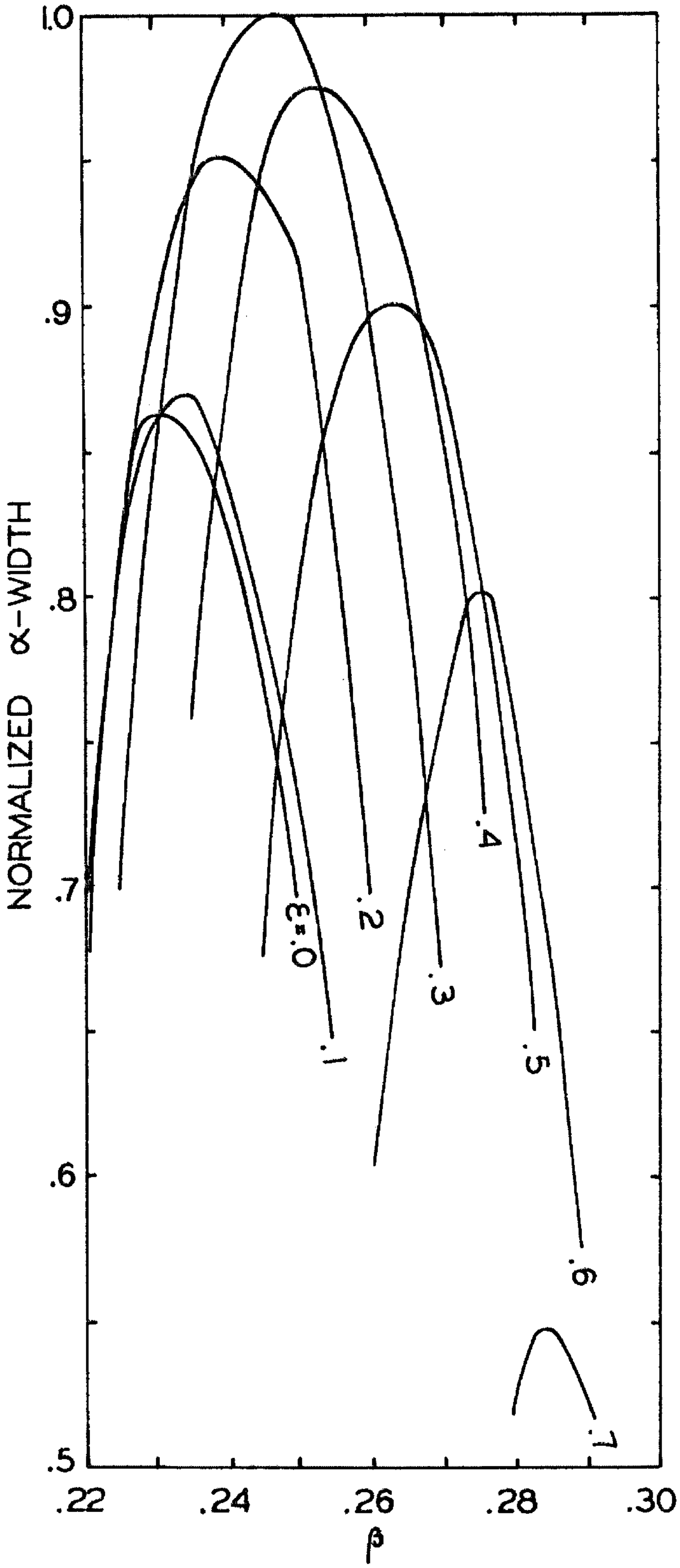

Figure 7. Plot of the relative α-width of the ovals derived directly from Figure 6 for several values of ε.

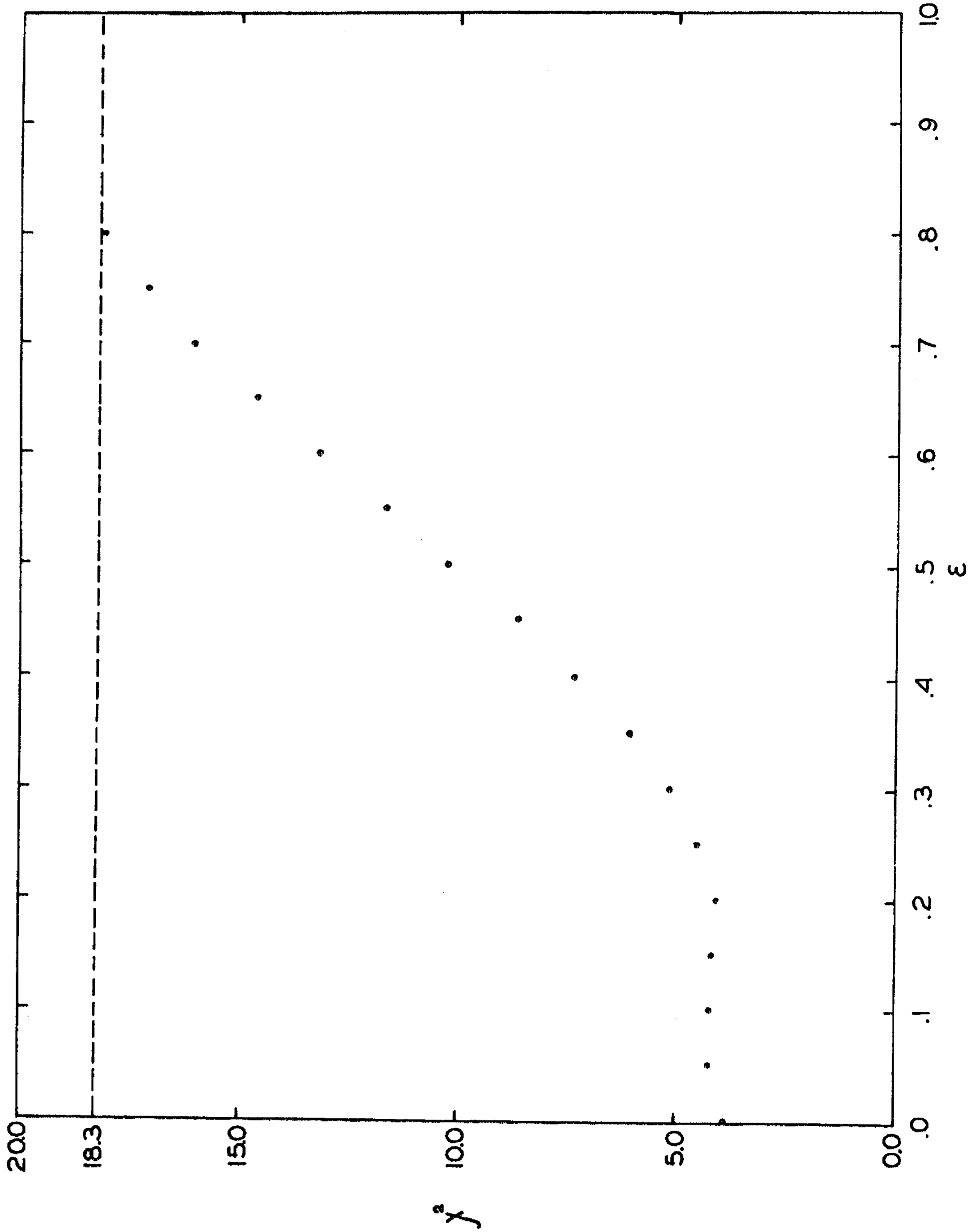

Figure 8. Plot of the discrete values of χ^2 indicating the goodness of fit as a function of ϵ.

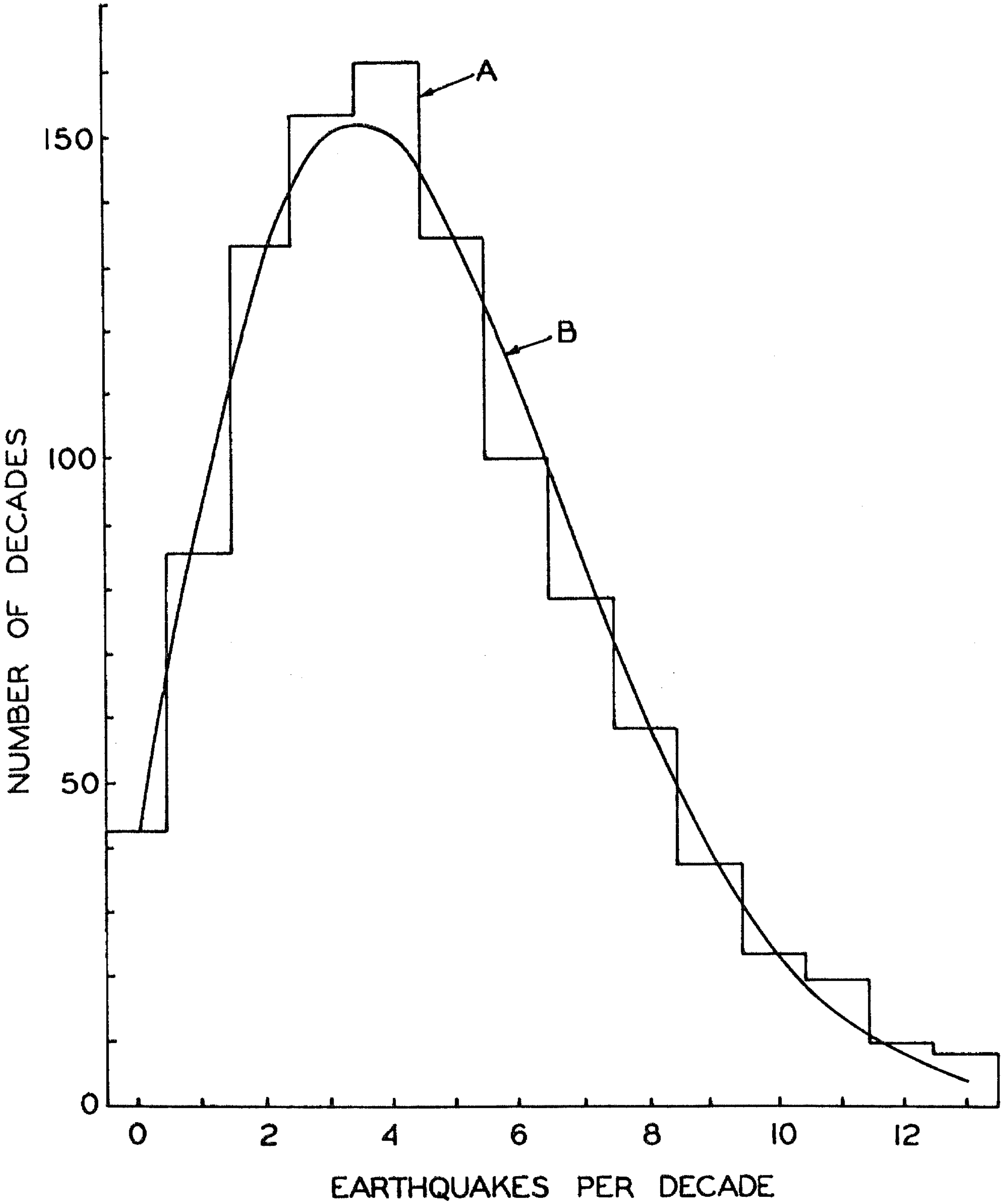

Figure 9. A) Histogram for all shocks with M≥3.0 (decades with more than 13 events and also decades immediately preceding and following them were rejected from the original sequence). B) Theoretica curve based on the model with the parameters $\alpha=0.17$, $\beta=0.24$, $\epsilon=0$ and $\chi^2=3.875$.

the χ^2 test shows that the biggest value of ϵ which is still acceptable is slightly greater than .80 (Figure 8). The plot in Fig. 7 for the relative normalized α width suggests that the fit of $p_\alpha(\eta)$ to the histogram in Fig. 9 becomes worse when ϵ is greater than .60 to .70.

SOME POSSIBLE PERTURBATION GENERATING MECHANISMS AND THEIR RELATION TO THE MODEL

In the formulation of the model for triggering earthquakes, the assumption was made that perturbations exist. A possible source of energy that might generate such perturbations was not considered at that time. In this section attention will be brought back to this question in order to see which are acceptable mechanisms and ways to generate them. Furthermore, some of the possibly important properties of these mechanisms will be pointed out, as an indication whether or not a chosen system may indeed fall into the general class of acceptable perturbation mechanisms.

As was pointed out, many authors have considered the idea of a small perturbation being superimposed on the slowly varying principal driving mechanisms that cause the earthquakes. The most frequently considered perturbations have been solid earth and ocean tides.

Solid earth tides are the deformation of the whole earth caused by the gravitational fields mainly of the sun and moon. Although the mass of the sun is many times greater than that of the moon, the tide producing force of the moon is 2.17 times greater than the solar one. This is because of the much smaller distance between the earth and the moon then between the earth and the sun. Because of the earth's rotation about the sun and its own axis and because of the moon's rotation about the earth, gravitational fields that generate tides are continually changing. The biggest tidal forces occur in the syzygy positions, i.e., during the new moon and the full moon, when the sun, earth and the moon are located along a straight line. The perfect straight line configuration occurs every 18.6 years, but an approximate configuration along the straight line will occur twice a month. In the quadrature positions, i.e., during the first and the last quarter moon, the sun and the moon are at 90° as observed from the earth. During this time the tides are almost 2.5 times smaller then in the syzygy positions. In addition, the tidal forces are effected by the eccentricity of the moon's orbit, which accounts for about 30% force difference between perigee and apogee positions. Especially big tides will occur during the periods when the perigee and the syzygy positions are closely spaced in time. This occurs approximately every 17.7 years. The surface amplitude of the earth tide can reach 30-50 cm measured from the geoid surface and the normal to the geoid can deflect by .02 inches. This effects the force of gravitation by about .3 mgals.

The shortest period of tidal variations is close to 12 hours, and this is considerably greater than the fundamental period of free oscillations of the whole earth which is of the order of one hour. Thus, it is often assumed that the earth responds to the driving tidal forces in a steady manner, and for this reason earth tides are usually considered to be of the equilibrium type.

The tidal potential can be written in the form

$$U = b_w f(\theta, \lambda, t, g, k_a)$$

where b_w is the amplitude constant belonging to the variation of the frequency w, θ is latitude, λ is the longitude, t - time and g and k_a are gravitational and "general lunar" constants.

It will be examined now whether the tides can be thought of as a mechanism generating the perturbation forces. First, it may be observed that the tidal perturbations may be taken as stationary. The next requirement is that the width of the power spectrum given by ϵ falls in the permissible range of the ϵ values that were obtained for the perturbation model. It will be seen that the variations in the f(t) will be proportional to the variations in the tide potential for the fixed θ and λ proportional to the constant factor b_w. Munk and Macdonald (1960) have listed b_w for some of the largest tidal components. Fig. 10(a) shows their values for constant b_w plotted versus frequency of oscillations in cycles per day. On the same Fig. 10(a) the calculated value of $\epsilon = .431$ is given. Hoskinson (1951) made a harmonic analysis of the gravity observations at several stations including Pasadena. The result of his analysis which considered only the principal tidal components is plotted in Fig. 10(b). The corresponding value of ϵ becomes .487. If the α width criterion for the best fit is considered to be a meaningful indication of the best values of ϵ resulting from the histogram in Fig. 9, then ϵ close to .35 would seem to be the optimum. However, as was mentioned before, a good fit is permitted by all values of ϵ ranging from 0.00 to .50, or .60, and the

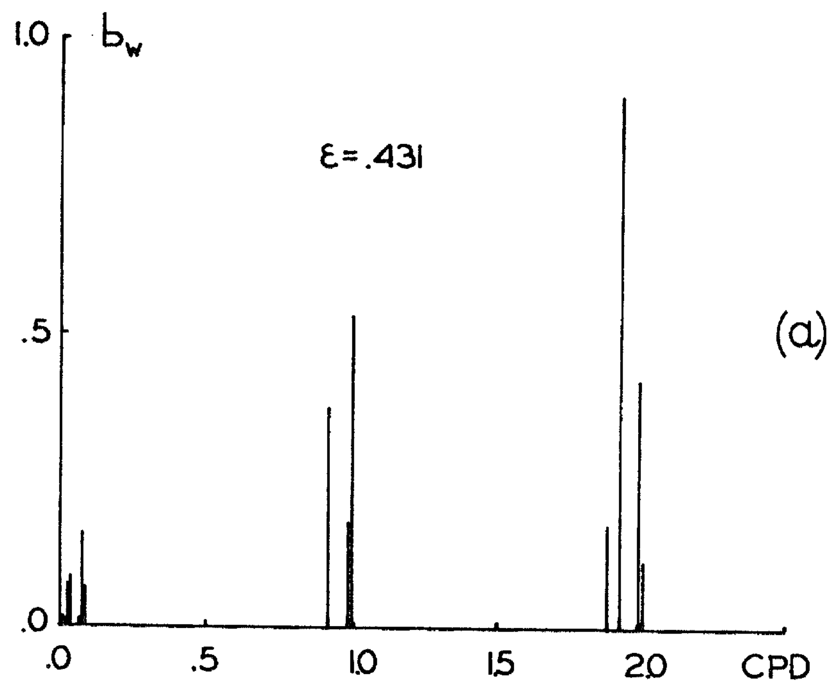

Figure 10a. The amplitude factor b_w for computing the potential of equilibrium tides (W.H. Munk and G.F. Macdonald, 1960)

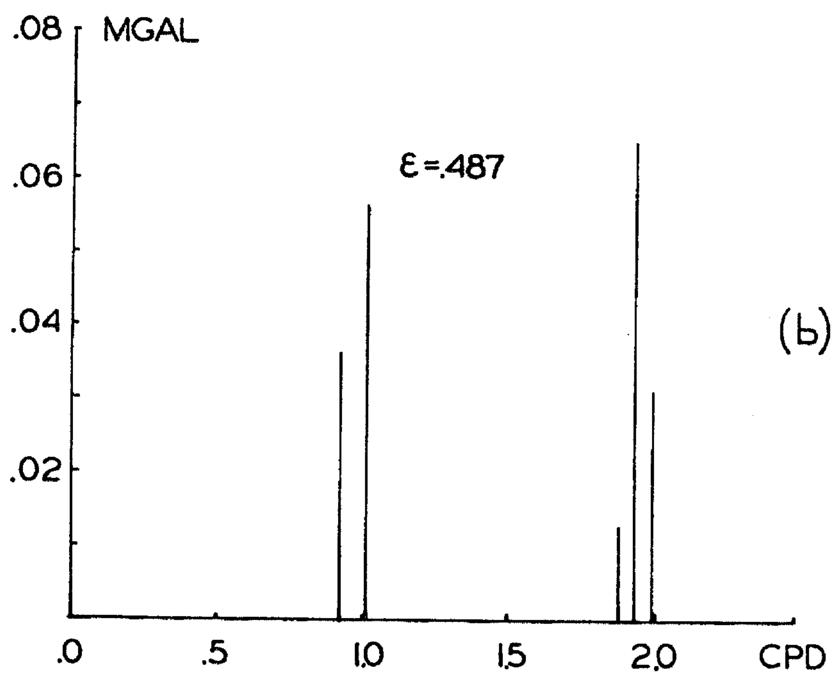

Figure 10b. Harmonic components of gravity observations at Pasadena (A.J. Hoskinson, 1951)

above values .431 and .487 fall in that range. Since the absolute amplitudes of the perturbation forces are not relevant for the model considered but only ϵ spectral width and stationarity in time, it may be concluded that in the light of the data and the present analysis, tidal perturbations may be accepted as a triggering mechanism. It should be remembered, however, that this is so if the basic assumptions leading to the perturbation model are acceptable and if the power spectrum of the gravity variation is proportional to the power spectrum of f(t) defined by the difference of the norms. Since there are many norms which will preserve this relationship, this restriction does not seem to be a serious one.

Experimental measurements of the earth tides usually do not contain results due to the solid earth tides alone. Many other effects are superimposed on them. This is in particular true for the deformations due to the ocean tides (e.g. John Kuo and Maurice Ewing, 1966) and in a lesser degree to the barometric pressure and temperature variations. Other fluctuations of geologic, tectonic, or meterological origin may also be significant. The manner in which the ocean tides are superimposed on the earth tides is usually very complex and nonuniform because of the variable boundary conditions and inhomogeneities, in particular along the continental shelves and the adjacent continental regions. However, periods of the ocean tidal variations are the same as those for the earth tides. Therefore, it may be expected that the change in the ϵ from these additional effects would not be very significant.

There are many other phenomena that are periodic in their character and so could represent the origin of the perturbation forces. Diurual and seasonal variations in temperature could be considered. However, as Jeffreys (1938) has pointed out, diurual variations in temperature are not effective deeper than two or three decimeters, and annual variations penetrate at most several meters. These depths are so small that it is hardly possible to imagine that the direct temperature fluctuations would cause any significant forces in the crust that would trigger earthquakes. If the temperature effect is indeed significant, it is probably generating the perturbation forces indirectly through some other mechanism, e.g. by influencing atmospheric variations, thus causing rapid changes in barometric pressure and winds.

Most of the other existing perturbations are either not stationary on the time scale considered in this analysis, or their amplitudes when compared to the tidal perturbations appear to be very small. Their primary, or secondary, contribution to the f(t) function can thus be neglected. Other possible time scales that might be used in the statistical analysis of the earthquake occurrence problem, such that most of the presently known perturbation generating mechanisms can be included in the final f(t), will not be considered here. The reason for this is that the interval of time during which accurate and reliable data exist is short on the relative geological scale and, therefore, no sound statistical results can be expected.

The statistical perturbation model does not require in its formulation any specific amplitudes of the perturbations, nor slopes of the gradually decaying $F_D^k(t)$ functions. It may be useful here to estimate some of these amplitudes and slopes in order to understand better the properties of the model. Brune (1968) made some speculative calculations of the rate of the average slip along the major fault zones. For the Southern California region he gives for the time period from 1934 to 1963:

Imperial Valley	3.2	cm/ye
San Andreas Fault	.03	"
Kern County	17.0	"
Total Area	5.8	"

Here, for the sake of the present discussion only, it can be assumed that the average rate of the slip in the whole of the Southern California is close to 6 cm/year. If this rate is on the average monotonic, it shows that the rate of the slip in a 10 day interval is of the order of .165 cm/10 days. Since the amplitude of the solid tide on the other hand is of the order of 10 cm, it follows that the monotonically decreasing functions $F_D^k(t)$ in Fig. 3 could be taken with nearly zero slopes. This then suggests that only the greater extrema of f(t) will be significant in triggering earthquakes and consequently that the best α should be closer to its upper permissible values. This completely qualitative argument appears to be in excellent agreement with the formal conditions for the best fit of the perturbation model to the data (Fig. 6, Fig. 7, Fig. 8).

Morgan, Stoner and Dicke (1961) have summarized some of the results on the magnitudes of the stress-producing mechanisms.

Some of these which may fall into the permissible class of perturbation functions for the present model, are given below:

Source of Stress	Stress in dyne/cm^2
Lunar Tidal Stress	3.4×10^4
Tidal Change in Sea Level Over Continental Shelf	6×10^4
Atmospheric Pressure Change	2×10^4

Brune (1968) has estimated the average stress operating during the earthquakes in the Imperial Valley, California. His results are that the average operating stress is 34 bars (bar = 10^6 dyne/cm^2) for $M = 7.0$ to less than 1 bar for $M = 3.0$. Thus, it may be seen that the average stress during the earthquake could be of the order of 10^2 to 10^3 times greater than the periodically changing tidal stress. However, because of uncertainties involved in these computations, the above order comparisons should be taken with caution.

CONCLUSIONS

Data from a period of 33 years on the occurrence of earthquakes in Southern California were used to present histograms showing the probability distribution of earthquakes. It is believed that these results may prove to be useful in other studies that are related to the general field of the earthquake statistics.

A simple perturbation model was analyzed with the aim of developing a simple but still fairly general physical model which could allow calculations of the statistical distribution as function of model parameters. It was shown that for the earthquakes studied, tidal perturbations are a triggering mechanism that is possible in a statistical sense. It should be emphasized that this conclusion is a statistical conclusion which is a consequence of accepting a H_0 hypothesis, which only says that the model chosen, together with all accompanying assumptions, is a permissible one. Therefore, the above does not exclude other models whose physical behavior could lead to some other distributions which might fit the earthquake data just as well as the distribution presented here.

APPENDIX I

In this appendix some results on the distribution of the maxima of a random function are briefly outlined following Cartwright and Longuet-Higgins, (1956) and Rice (in selected papers on Noise and Stochastic Processes, edited by Wax 1954).

Let

$$\xi_1 = f(t) = \sum_n c_n \cos(\sigma_n t + \epsilon_n) \tag{1a}$$

$$\xi_2 = f'(t) = -\sum_n c_n \sigma_n \sin(\sigma_n t + \epsilon_n) \tag{1b}$$

$$\xi_3 = f''(t) = -\sum_n c_n \sigma_n^2 \cos(\sigma_n t + \epsilon_n) \tag{1c}$$

Let $p(\xi_1, \xi_2, \xi_3)$ be the joint probability distribution of f, f', f''. The mean frequency of maxima of f(t) in the interval $\xi_1 < f < \xi_1 + d\xi_1$ is then

$$F(\xi_1)\, d\xi_1 = \int_{-\infty}^{0} \left[p(\xi_1, 0, \xi_3) \, |\xi_3| \, d\xi_1 \right] d\xi_3 \tag{2}$$

The total mean frequency of maxima is

$$N_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{0} p(\xi_1, 0, \xi_3) \, |\xi_3| \, d\xi_1 \, d\xi_3 \tag{3}$$

The probability distribution of maxima is found by dividing the mean frequency of maxima by the total mean frequency of maxima.

Since ξ_1, ξ_2 and ξ_3 are each the sum of an infinite number of variables with zero mean and random phase the joint probability distribution of (ξ_1, ξ_2, ξ_3) is normal by the central limit theorem (Rice, selected papers on Noise and Stochastic Processes, edited by Wax, 1954). The matrix of correlations is then

$$\begin{pmatrix} m_0 & 0 & -m_2 \\ 0 & m_2 & 0 \\ -m_0 & 0 & m_4 \end{pmatrix} \tag{4}$$

Defining

$$\Delta = m_0 m_4 - m_2^2$$

we have then

$$p(\xi_1, \xi_2, \xi_3) = \frac{1}{(2\pi)^{3/2}(\Delta m_2)^{1/2}} \exp\left\{-\frac{1}{2}\left[\frac{\xi_2^2}{m_2} + \left(m_4\xi_1^2 + 2m_2\xi_1\xi_3 + m_0\xi_3^2\right)/\Delta\right]\right\} \tag{5}$$

Substitution in (2) gives

$$F(\xi_1) = \frac{1}{(2\pi)^{3/2}(\Delta m_2)^{1/2}} \int_{-\infty}^{0} \exp\left\{-\frac{1}{2}\left(m_4\xi_1^2 + 2m_2\xi_1\xi_3 + m_0\xi_3^2\right)/\Delta\right\}|\xi_3|\,d\xi_3 \tag{6}$$

Defining

$$\frac{\xi_1}{m_0^{1/2}} = \eta \quad \text{and} \quad \frac{\Delta^{1/2}}{m_2} = \delta \tag{7}$$

and evaluating the integral (6) gives

$$F(\xi_1) = \frac{1}{(2\pi)^{3/2}} \frac{\Delta^{1/2}}{m_0 m_2^{1/2}} e^{-\frac{1}{2}\eta^2}\left[e^{-\frac{1}{2}\frac{\eta^2}{\delta^2}} + \left(\frac{\eta}{\delta}\right)\int_{-\eta/\delta}^{\infty} e^{-\frac{1}{2}x^2}dx\right] \tag{8}$$

Because of (7) the probability distribution of η is $m_0^{1/2}$ times the distribution of ξ_1 i.e.

$$p(\eta) = m_0^{1/2} p(\xi_1) = \frac{m_0^{1/2} F(\xi_1)}{N_1} \tag{9}$$

From (3) N_1 may be found to be

$$N_1 = \frac{1}{2\pi} \left(\frac{m_4}{m_2} \right)^{1/2} \tag{10}$$

and then finally

$$p(\eta) = \frac{1}{(2\pi)^{1/2}} \left[\epsilon e^{-\frac{1}{2} \frac{\eta^2}{\epsilon^2}} + (1-\epsilon^2)^{1/2} \eta e^{-\frac{1}{2}\eta^2} \int_{-\infty}^{\eta\frac{(1-\epsilon^2)^{1/2}}{\epsilon}} e^{-\frac{1}{2}x^2} dx \right] \tag{11}$$

with

$$\epsilon^2 = \frac{\delta^2}{1+\delta^2} = \frac{\Delta}{m_0 m_4} = \frac{m_0 m_4 - m_2^2}{m_0 m_4} \tag{12}$$

ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to Professor D.E. Hudson for encouragement and assistance during this work. I am also grateful to Professors C.H. Dix and G.W. Housner for critically reading the manuscript and offering many valuable suggestions. Thanks are expressed to Professor J.N. Brune for many valuable discussions.

This study was supported by the National Science Foundation grant(Engineering Mechanics Program).

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Department of Engineering and Applied Science
California Institute of Technology